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Shape Optimization by Using Simulated Biological Growth Approaches

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The shape optimization of structures by using simulated biological growth approaches is described in this paper. These approaches simulate the behavior of adaptive shapes of biological structures by growth and atrophy with respect to the natural loading applied. Improved design procedures are proposed to extend the capability of the approaches for shape optimization problems. Some numerical results are presented to demonstrate the usefulness of proposed methods.

Introduction

THE failure of machine structures under fatigue loading generally occurs at sites of high local notch stress. Therefore, it is interesting for the designer to develop effective methods of optimizing the shape of the machine structures for reducing the stress peaks. Many shape optimization techniques were proposed.[1-3] However, biological structures such as bones and trees, which change their own shape by growth and atrophy to adapt to external loads for reducing stress peaks, provide a natural and simple example for shape optimizations. Recently, many simulated biological growth approaches for shape optimizations have been proposed.[4,5] The idea proposed in Ref. 4 is to simulate the growth ring in trees by covering the finite element mesh with a thin layer of finite elements having a much smaller Young's modulus than the material below. The approach proposed by Azegami[5] is to optimize domain shapes by growth in all parts of the domain. A shape optimal method[6] based on fictitious loads acting on an auxiliary structure and using the resulted deformation to update the shape is similar to simulated biological growth approaches in the procedure. However, its fictitious loads are not based on the biological growth law. In this paper, the simulated biological growth approaches for shape optimization of machine structures is presented. Improved design procedures of the simulated biological growth approaches for shape optimization are proposed and demonstrated with numerical examples.

Simulated Biological Growth Approaches

The procedure of the simulated biological growth approach[4,5] is based on the iterative process of two finite element analysis steps. The first one is finite element analysis of a static load case to get a stress distribution over the whole structure. The other is an incremental growth analysis based on a growth law in which the swelling strain is defined. The growth law is to calculate the swelling strain ϵ_T on each node point as follows:

$$\epsilon_T = \left(\frac{\sigma_{\rm von} - \sigma_{\rm bas}}{\sigma_{\rm bas}}\right) \cdot h \tag{1}$$

where h is the incremental growth rate and $\sigma_{\rm von}$ and $\sigma_{\rm bas}$ are the von Mises stress in this node and the basic stress, respectively. The swelling strain is transferred to a fictitious temperature field that is applied in a further finite element analysis as the only applied loading.

$$\epsilon_T = \alpha \Delta T \tag{2}$$

$$\text{TEMP} = T_{\rm ref} + \Delta T \tag{3}$$

where α is the heat expansion coefficient, ΔT is the equivalent difference temperature of the swelling strain, $T_{\rm ref}$ is the reference temperature, and TEMP is the temperature loading on each node. The boundary condition in this step is changed to a different form according to the restriction of shape deformation for design.

The results of the second finite element analysis are the incremental displacements and add them to the coordinates of the first step. These updated coordinates are used to construct a new structural shape. The termination of the iterative process is judged by the convergence of the criteria. The flowchart of the simulated biological growth approaches for shape optimization is illustrated in Fig. 1.

According to the previous study,[7] varying the values of the incremental growth rate h affects the convergence rate. Furthermore, increasing or decreasing the value of basic stress $\sigma_{\rm bas}$ decreases or increases the value of the area, respectively.

Improved Design Procedures

Currently, simulated biological growth approaches can only be used for designing the shape of machine structures to reduce local stress peaks. Because the change of the value of basic stress affects the value of area, it seems that the simulated biological growth approach can also be applied to the

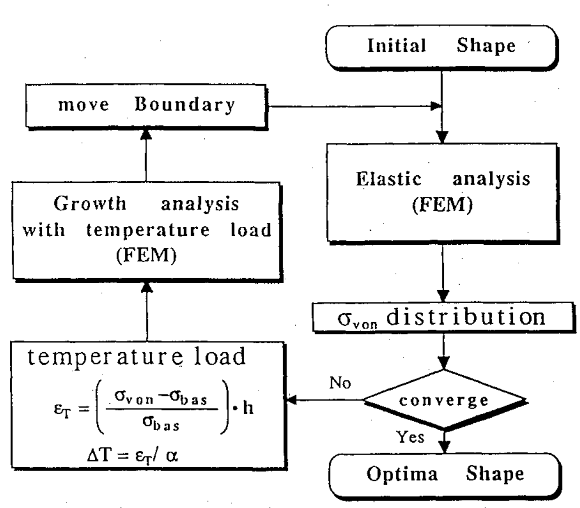

Fig. 1 Flowchart of the simulated biological growth approach.

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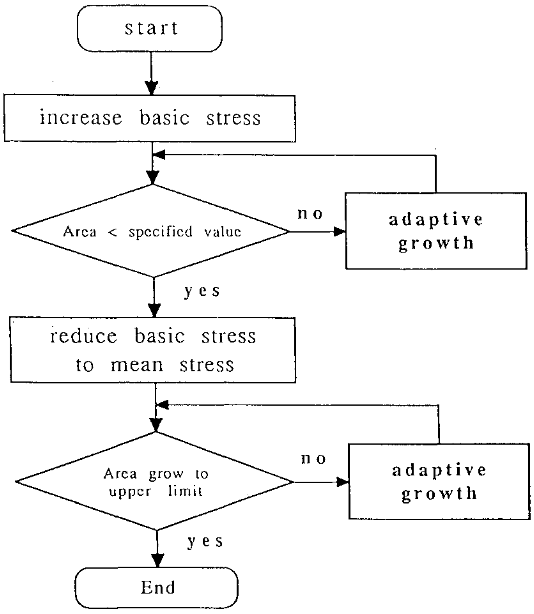

Fig. 2 Flowchart of the two-stage improved design procedure 1.

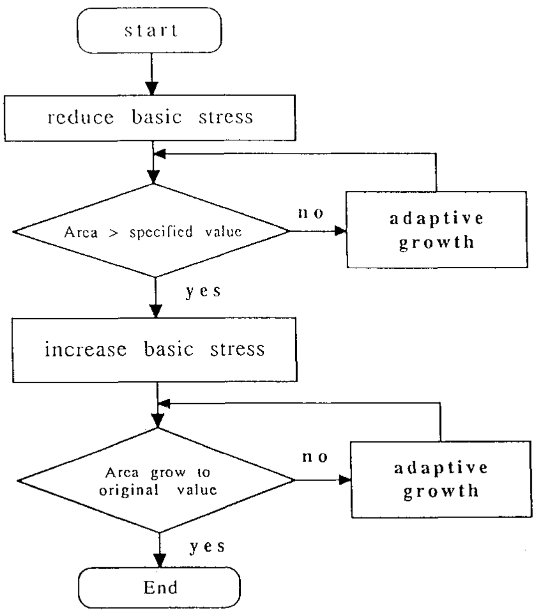

Fig. 3 Flowchart of the two-stage improved design procedure 2.

shape optimization problems subject to area constraint or to minimize the area.

For shape optimization problems that were designed for minimum area subject to the maximum stress constraint, an improved design procedure is proposed. The flowchart of this improved approach is the same as shown in Fig. 1. However, the value of the basic stress is much higher than the value of the average von Mises stress of initial design for the purpose of reducing the value of area. Furthermore, termination criteria in this improved approach is replaced by the value of maximum allowable stress.

Other improved design procedures for the shape optimization problems that were designed for minimizing the maximum stress subject to the area constraint are also proposed by using a two growth step approach, as illustrated in Figs. 2 and 3. In these improved design procedures, two adaptive growth steps are involved during shape optimization. Each adaptive growth step contains the first finite element analysis and the incremental growth analysis procedures as described in the previous section. The first growth step uses a higher (or lower) basic stress than the value of the average von Mises stress of initial design (mean stress) to decrease (or increase) the value of area. The second growth step changes the basic stress to the mean stress for adjusting the value of area to its limit. During the design procedures, the value of area becomes the termination criteria.

Numerical Results

Three examples are presented to demonstrate the efficiency of using simulated biological growth approaches for shape optimization of structures. The MSC/NASTRAN finite element program (version 67A) was selected to perform the finite element analysis. All numerical results are performed on an SGI 4D/310 VGX workstation.

Cantilever Beam Under Tip Shear Loading

A cantilever beam under tip uniform distributed shear loading, as shown in Fig. 4, is chosen as the first example. The length and the width of the beam are 5 m and 1.2 m, respectively. The value of top shear loading is 6 MN/m. The value of Young's modulus and Poisson's ratio are 210 GPa and 0.3, respectively.

The domain was discretized into 150 four-node rectangular elements. The objective function is defined as the ratio of $\sigma_{max}/\sigma_{bas}$. The σ_{max} is the maximum von Mises stress in the problem domain. The values of incremental growth rate h and basic stress σ_{bas} are defined as 0.05 and the value of average von Mises stress of initial design, respectively. The iteration history and the history of the stress value are shown in Figs. 5 and 6, respectively. The maximum and average stresses in Fig. 6 are the maximum von Mises stress in the whole problem

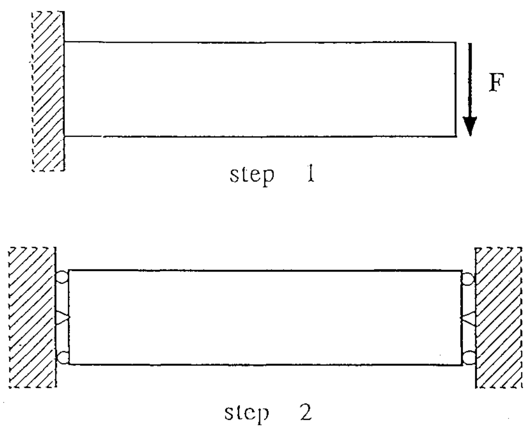

Fig. 4 Cantilever beam under tip shear load.

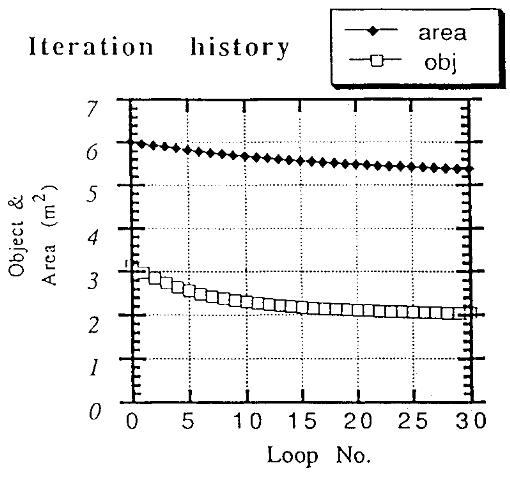

Fig. 5 Iteration history of the beam problem.

domain and the average von Mises stress of the current design, respectively. The initial and optimum shapes of this problem are shown in Fig. 7.

Square Plate with a Hole Under Biaxial Tension

The second example is a square plate with a hole subjected to uniform in-plane tensile loads along its edges, as shown in Fig. 8. The stress concentrations occur at the vertices of the hole. The optimization objective is therefore to find the shape of the hole that minimizes the stresses in the boundary hole elements. The length of the plate and hole are 12 in. and 2 in., respectively. The value of Young's modulus, Poisson's ratio, and load P are 30×10^6 psi, 0.3, and 10 lb/in., respectively.

Due to symmetry, only a quarter of the plate is modeled, as shown in Fig. 9.[8] The domain was discretized into 30 four-node rectangular elements. The objective function is also defined as the ratio of $\sigma_{\max}/\sigma_{\text{bas}}$. The value of h and σ_{bas} are defined as 0.25 and the value of average von Mises stress of initial design, respectively. The iteration history and the history of the stress values are shown in Figs. 10 and 11, respectively. The initial and optimum shape of this problem are shown in Fig. 12.

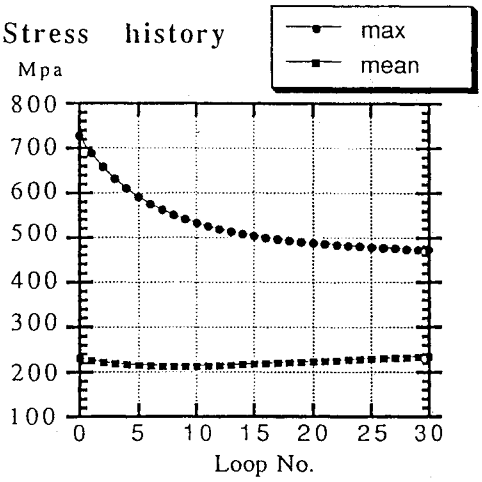

Fig. 6 History of maximum and mean stress for the beam problem.

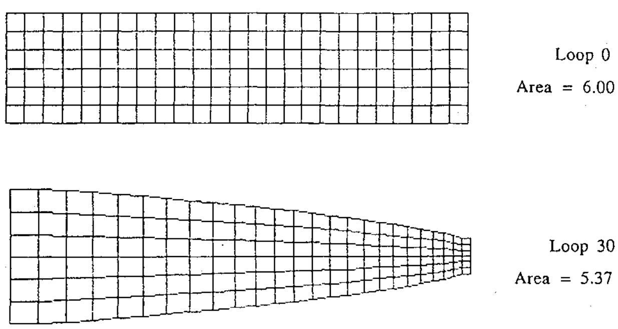

Fig. 7 Initial and optimum shapes of the beam problem.

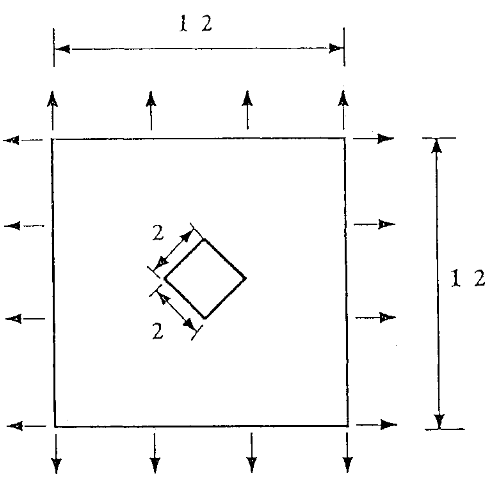

Fig. 8 Square plate with hole under biaxial loading.

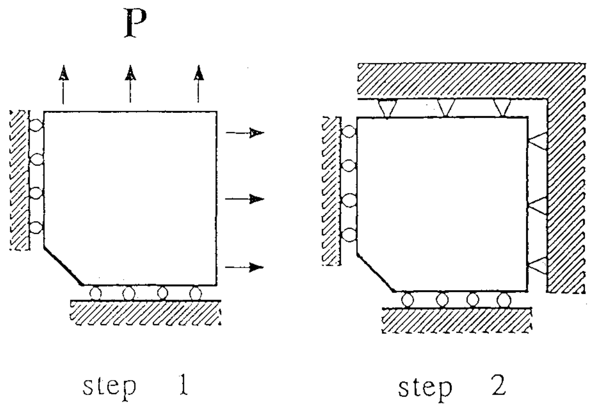

Fig. 9 Shape optimal design model for the plate problem.

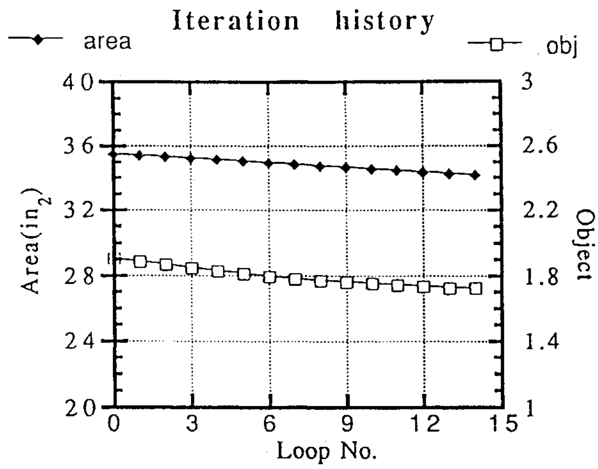

Fig. 10 Iteration history of the plate problem.

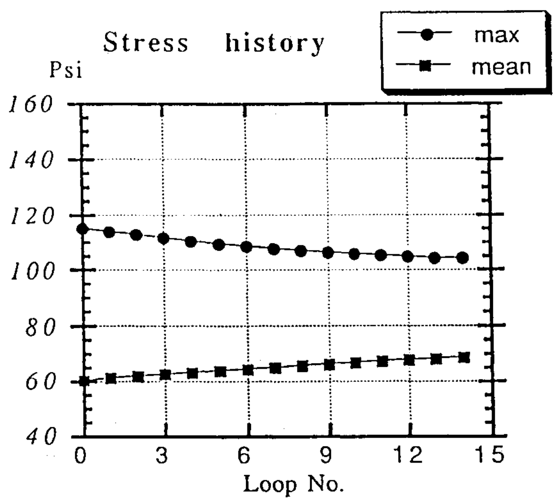

Fig. 11 History of maximum and mean stress for the plate problem.

Column Under Top Compressive Loading and Gravity

A column under top compressive loading and gravity is selected as the third example problem. Due to symmetry, half of the column was analyzed, as shown in Fig. 13. The length

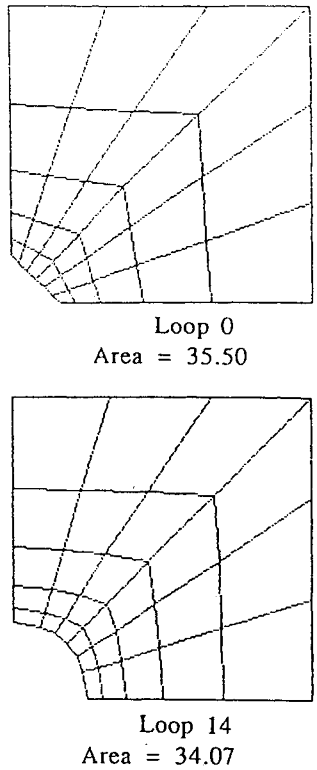

Fig. 12 Initial and optimum shapes of the plate problem.

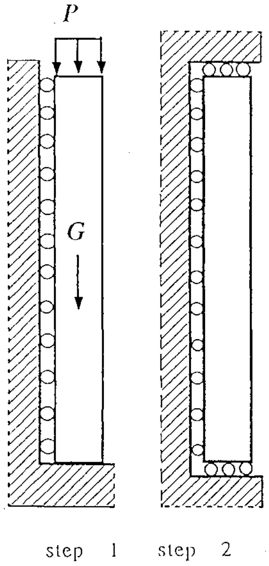

Fig. 13 Column under top load *P* and gravity *G* in a symmetric half.

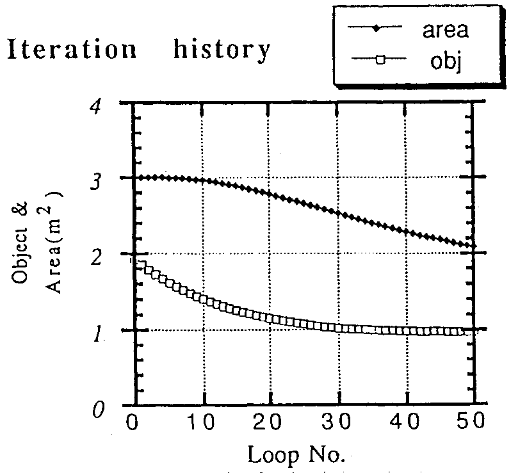

Fig. 14 Iteration history of the column problem.

and the width of the column are 5 m and 1.2 m, respectively. The value of top compressive loading is 60 kN/m. The density, Young's modulus, gravity acceleration, and Poisson's ratio of this column are 7.86 kg/m^3, 210 GPa, 9.8 m/s^2, and 0.3, respectively.

The domain was discretized into 75 four-node rectangular elements. In the beginning, the value of incremental growth rate h was defined as 0.05. The value of average von Mises stress of initial design, 0.25 MPa, was chosen as the value of basic stress σ_{bas}. The object function is defined as the ratio of $\sigma_{max}/\sigma_{bas}$. The iteration history is shown in Fig. 14. The history of the stress values is shown in Fig. 15. Figure 16 shows the initial and optimum shape of this problem.

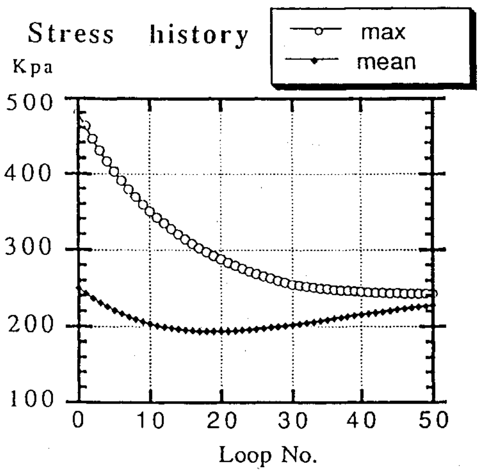

Fig. 15 History of maximum and mean stress for the column problem.

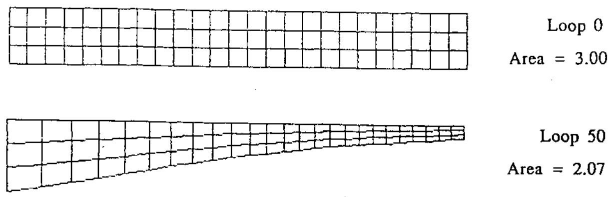

Fig. 16 Initial and optimum shapes of the column problem.

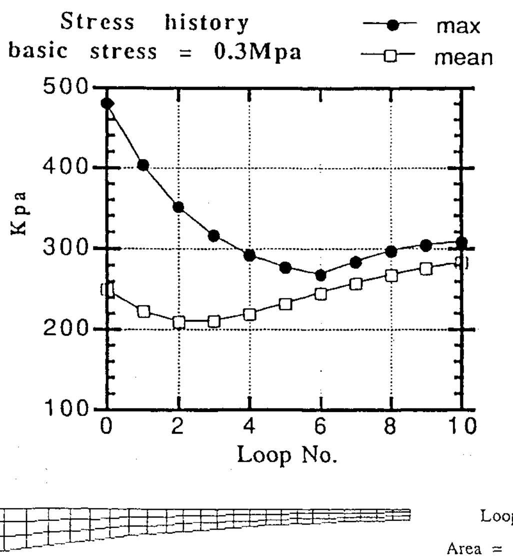

Fig. 17 History of stress and shape of the column problem with σ_{bas} = 0.3 MPa.

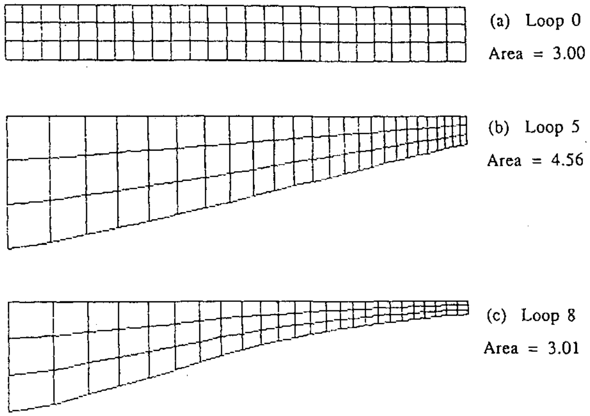

Fig. 18 Initial and optimum shapes of the column problem with constant area constraint.

Table 1 The numerical values of the column problem with constant area constraint

	σ_{bas}	Loop no.	Area	σ_{max}
Initial		0	3.00 m^2	480 Kpa
Stage 1	180 Kpa	5	4.56 m^2	256 Kpa
Stage 2	250 Kpa	8	3.01 m^2	233 Kpa

Column Problem with Minimum Area Objective Function

A fourth example problem is the column problem that was designed for minimum area subjected to maximum stress constraint $\sigma_{all} = 310$ KPa. The first improved design procedure was employed for this problem. The basic stress σ_{bas} was increased to 0.3 MPa. The value of maximum stress was selected as the convergence criteria. By setting the value of h equal to 0.25, the history of the stress values and the shape at the 10th iteration of this case is shown in Fig. 17.

Column Problem with Constant Area Constraint

The last example problem is the column problem that was designed to minimize the maximum stress subjected to constant area constraint. The improved design procedure, as shown in Fig. 3, was selected for this case. The value of σ_{bas} was defined as 180 KPa during first stage. The second stage increased the value of σ_{bas} to 250 KPa. The numerical results of this case are shown in Table 1 and Fig. 18.

Conclusions

Shape optimization of structures by using the simulated biological growth approaches with a fictitious temperature loading is presented. Some improved design procedures are presented to extend the capability of the simulated biological growth approaches for dealing with the shape optimization problems trying to minimize area or subject to area constraint. Several numerical examples were demonstrated to illustrate the effectiveness of the proposed approaches.

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